

## TRANSPIRATION DRYING OF POROUS HYGROSCOPIC MATERIALS

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**Abstract**—The phenomenon of transpiration drying of porous hygroscopic bodies has been investigated, both experimentally and theoretically, to understand the process of heat and mass transfer involved in the drying.

The experiments consist of measuring the drying rates of thin sheets of paper dried by passing dry air through the sheets. The time history of drying was measured, under different air flow conditions and for different paper thicknesses, by a gravimetric method. The drying rates were deduced from these data and correlated against flow variables. Characteristic constant and falling drying rates were observed.

A two region theoretical analysis of the problem is made. In the first phase, a bulk analysis of the constant drying regime is worked out. This analysis provides a means of properly defining the heat- and mass-transfer coefficients. The analysis is extended to include a specific internal evaporative mechanism within the fibrous body. Analytical and numerical solutions in the form of an evaporative coefficient have been obtained. In the second phase of the analysis, the falling drying rate regime is accounted for by the fact that the vapor pressure inside the body decreases with decreasing moisture content. A numerical solution to the transient mass-energy equation is obtained (incorporating the effects of solid matrix conduction and fiber shrinkage) to predict the drying rate as a function of time. The results agree well with observed data.

Based on the analytical and experimental results correlations are given to predict the time to dry from a given initial moisture content to a specific final moisture content.

### NOMENCLATURE

<p><math>a, b</math>, constants in the linearized Clausius-Clayperon equation (equation 20);</p> <p><math>A</math>, constant in the Clausius-Clayperon equation for saturated vapor pressure [atm];</p> <p><math>C</math>, specific heat (subscripted) [J/kg K];</p> <p><math>D</math>, diffusion coefficient for water vapor in air [<math>\text{m}^2/\text{s}</math>];</p> <p><math>E</math>, coefficient of evaporation (equation 22);</p> <p><math>\mathcal{E}</math>, moisture evaporation flux based on the nominal area of the porous body, normal to the air stream [<math>\text{kg}/\text{m}^2\text{s}</math>];</p> <p><math>f</math>, local moisture fraction within the porous body, <math>= m/\rho_S</math>;</p> <p><math>F</math>, total moisture fraction, <math>= (\text{mass of moisture in the body})/(\text{mass of dry body})</math>;</p> <p><math>G</math>, mass flux of dry air (based on nominal area) [<math>\text{kg}/\text{m}^2\text{s}</math>];</p> <p><math>h</math>, heat-transfer coefficient (based on nominal area) [<math>\text{J}/\text{m}^2\text{s K}</math>];</p> <p><math>h_m</math>, mass-transfer coefficient (based on nominal area) [<math>\text{m}/\text{s}</math>];</p> <p><math>H</math>, volumetric (internal) heat-transfer coefficient [<math>\text{J}/\text{m}^3\text{s K}</math>];</p> <p><math>H_m</math>, volumetric mass-transfer coefficient [<math>\text{s}^{-1}</math>];</p> <p><math>I</math>, specific enthalpy (subscripted) [<math>\text{J}/\text{kg}</math>];</p>	<p><math>j_h</math>, nondimensional heat-transfer coefficient;</p> <p><math>j_m</math>, nondimensional mass-transfer coefficient;</p> <p><math>k</math>, Darcy permeability of dry porous body defined by <math>k = \sqrt{(Gv_{\text{air}})}</math> (mean pressure gradient across the dry body) [<math>\text{m}</math>];</p> <p><math>K</math>, thermal conductivity (subscripted) [<math>\text{J}/\text{m K s}</math>];</p> <p><math>l</math>, upstream boundary layer thickness (subscripted) [<math>\text{m}</math>];</p> <p><math>L</math>, thickness of dry porous body [<math>\text{m}</math>];</p> <p><math>m</math>, mass of moisture per unit volume of porous body [<math>\text{kg}/\text{m}^3</math>];</p> <p><math>P</math>, partial vapor pressure [<math>\text{N}/\text{m}^2</math>];</p> <p><math>q</math>, heat transferred from air to the body (based on nominal area) [<math>\text{J}/\text{m}^2\text{s}</math>];</p> <p><math>Q</math>, dimensionless heat transferred from air to the porous body, <math>= \frac{q}{G\lambda}</math>;</p> <p><math>r</math>, relative humidity [vapor pressure ratio];</p> <p><math>s</math>, specific humidity (mass of water vapor in unit mass of dry air);</p> <p><math>t</math>, time [<math>\text{s}</math>];</p> <p><math>T</math>, temperature [<math>\text{K}</math>];</p> <p><math>T^*</math>, characteristic temperature of the problem <math>= \lambda/C_a</math> [<math>\text{K}</math>];</p> <p><math>y</math>, distance through the thickness of the porous body [<math>\text{m}</math>].</p>
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### Nondimensional numbers

<p><math>Pe_L</math>, flow Peclet number, <math>= \frac{GC_aL}{K_a}</math>;</p> <p><math>Pe_k</math>, Peclet number based on Darcy permeability, <math>= \frac{GC_a k}{K_a}</math>;</p>	
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$$Pe_D, \text{ mass Peclet number, } = \frac{Gk}{D\rho_a^{\text{mean}}};$$

$$Pe_S, \text{ Peclet number based on conductivity of dry solid, } = \frac{GkCa}{K_S};$$

$$Le, \text{ Lewis number, } = \frac{K_a}{\rho_a^{\text{mean}}C_a D}.$$

#### Greek letters

- $\beta$ , ratio of specific heat of a substance to the specific heat of dry air (subscripted);
- $\delta$ , a quantity defined in equation (20);
- $\Delta$ , a quantity defined in equation (20);
- $\epsilon$ , porosity of the porous body (generally a function of the moisture content);
- $\eta$ , nondimensional distance through the thickness of the porous body,  $= y/L$ ;
- $\theta$ , nondimensional temperature (subscripted),  $= T/T^*$ ;
- $\lambda$ , latent heat of vaporization [J/kg];
- $\mu$ , ratio of molecular weights of vapor and air;
- $\rho_a^{\text{mean}}$ , dry air density at arithmetic means of inlet and outlet temperatures;
- $\rho$ , mass density (subscripted) [kg/m<sup>3</sup>];
- $\sigma$ , area density of dry porous body [kg/m<sup>2</sup>];
- $\tau$ , nondimensional time,  $= \frac{Gt}{\sigma_p}$ .

#### Subscripts

- $a$ , dry air;
- $p$ , paper;
- $S$ , solid matrix of the porous body (dry);
- $s, T$ , to represent vapor diffusion and thermal diffusion;
- $v$ , vapor;
- $w$ , water, wet bulb;
- $v, a$ , vapor in air;
- $v, w$ , vapor over water.

#### Superscripts

- $i$ , inlet condition (far upstream of body);
- $o$ , outlet condition;
- $s$ , inlet surface of the porous body;
- sat, saturated;
- $\dot{\phantom{x}}$ , the period over any symbol denotes the time rate of change of the quantity denoted by the symbol.

### INTRODUCTION

THE TRANSPIRATION drying of paper appears to have an advantage compared to the conventional hot surface drying because of the effective utilization of all the internal surface area of the fibrous structure for the exchange of heat and mass with the transpiring fluid (generally hot air). Lewis [1] and Sherwood [2] were the first investigators to analyze the drying of porous media. Sherwood discusses the drying data for the drying of clay, soap, sand, etc., under the influence or air flow past bodies made of the above materials.

The observation of three distinct regimes of drying, viz. constant drying rate regime, first falling rate and the second falling rate regimes, are reported. The constant rate phenomenon is a consequence of pure liquid evaporation and the falling rates of drying are caused by the decrease in the surface area for evaporation and the resistance to the moisture vapor flow inside the porous body. Following Sherwood, several workers have studied this problem by considering various types of heating and boundary conditions, including such physical phenomena as the hygroscopicity, heat pipe effects, etc.

In a different category of studies, several workers in the wool, textile, and chemical industries (Nordon [3], Gamson *et al.* [4]) have studied the problem of transpiration exchange of heat and mass between a fluid and a porous medium. Of particular interest to the present study are the works of Chu [5], Walser and Swenson [6], and Hodges [7] on the transpiration drying of paper. In certain respects the experimental works of Chu, and Walser and Swenson parallels the experimental work of this study while our theoretical analyses are similar to those attempted by Hodges. However, Hodges does not consider the falling rate regime in its entirety nor does his model contain the effects of thermal conduction and mass diffusion in the air stream both of which prove to be important.

### THE EXPERIMENTS

The details of the apparatus, its design and the experimental procedure are given in [8]. Only essential details are described here. The basic principle of the experiment is the measurement of the weight of a piece of paper while drying. The drying is caused by dry air flowing through the paper. A special cylindrical "paper holder" (made of 50 mesh copper screen and brass end sections) is designed to hold the paper against the pressure drop caused by the air flow through it. The paper holder also facilitates the measurement of the weight of the paper.

The paper holder on which paper is formed from a slurry of paper fibers is suspended in the drying chamber, through two air bearings, from one arm of a chemical balance. The deflections of the balance are transduced into electrical signals and recorded on a strip chart recorder. The temperature of the air inside the chamber, the air temperature at the exit section and the pressure drop across the paper are measured. The air is dried by calcium sulphate in series with silica gel and is heated to the desired test temperature. A schematic diagram of the important features of the experimental setup is given in Fig. 1. In each experiment the following are recorded:

- (i) A time history of the paper weight.
- (ii) The air flow rate and temperature just before inlet to paper.
- (iii) Air exit temperature after passing through the paper.
- (iv) Pressure drop (across the paper) history with time.

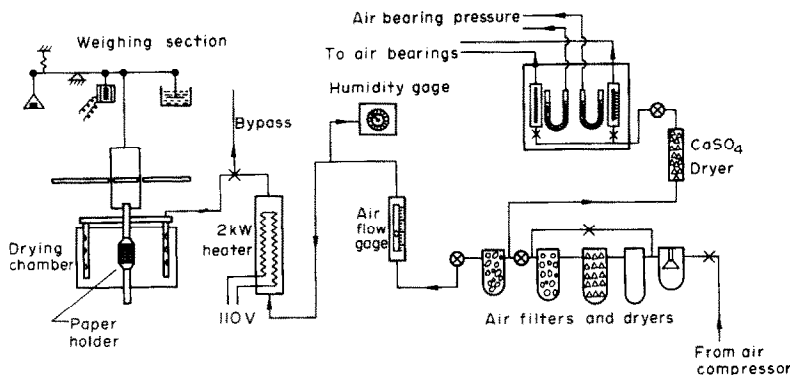


FIG. 1. Details of the drying chamber and the air supply.

(v) Dry paper weight and thickness (measured at the end of the test).

The experimental range included air flow rates between 31.7 kg/m<sup>2</sup>.s to 165 kg/m<sup>2</sup>.s (velocity through the paper 0.265 m/s to 1.375 m/s) and paper thickness from 72.5 to 300 μ (area density of paper 1.3–6.5 kg/m<sup>2</sup>). Most of the experiments were conducted with dry air at room temperature (18–22°C) though some runs were made with hot air (80°C). The porosities of paper sheets ranged from 0.86 to 0.89. The drying data was reproducible within the noise level in the measurement (maximum noise to signal ratio = 0.1). The “noise” was caused by the aerodynamic action of air flow through the paper as well as the flow of air in the bearings. During the reduction of the drying data, corrections were made for the time lag in the measuring system (spring dashpot system in the chemical balance beam). The pressure drop data has better reproducibility and very little noise.

In all of the experiments drying was observed to be in three distinct phases. The first was a very rapid initial transient in which the paper cooled down and the drying rate reached a maximum. In the second phase this maximum drying rate remained constant (“constant drying regime”) and finally a “falling rate” period in which the drying rate slowly changed from its maximum to zero. The duration of the third phase was the same order of magnitude as the second phase.

**THEORETICAL FORMULATION**

The theoretical models are based on the following assumptions:

1. The moisture removal is by pure evaporation. That is, no provision is made for moisture removal by mechanical action caused by the applied pressure drop across the paper.
2. Local thermodynamic equilibrium exists.

*1. Bulk analysis for steady state evaporation*

In this model, the constant rate evaporation phenomenon (constant drying) for evaporation from a porous body–moisture system is analyzed. The analysis uses only the conservation equations of mass and energy. The heat and mass transfer coefficients are defined and calculated from experimental data.

With reference to Fig. 2, we write the following conservation equation assuming that, (i) the mass flux of dry air entering the body is held constant and,

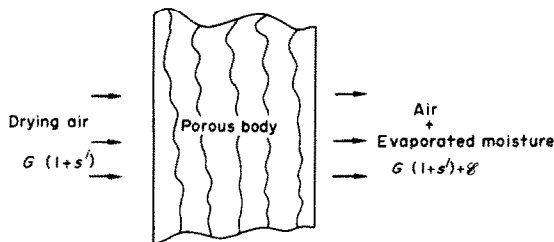


FIG. 2. Model for the porous body.

(ii) the temperature of the body throughout the thickness is uniform and equal to the wet bulb temperature of the inlet air.

Moisture Balance:

$$G(s^o - s^i) = \mathcal{E}. \tag{1}$$

Energy Balance (for time interval Δt and unit nominal area):\*

$$G(I_a^i + s^i I_v^i) \Delta t + M I_w(T_w) = (M - \mathcal{E} \Delta t) I_w(T_w) + G(I_a^o + s^o I_v^o) \Delta t \tag{2}$$

Energy inlet in the form of wet air enthalpy.	Sensible heat of liquid in a unit nominal area.	Sensible heat in the liquid remaining.	Enthalpy out flow in the outlet moist air.
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\*I is used for specific enthalpy instead of h to avoid confusion with a heat-transfer coefficient.

If we assume that the liquid is saturated at temperature  $T_w$  and noting that  $T_w = T_w^i$ , we obtain:

$$(T_a^i - T_a^o) = \frac{\lambda + C_v(T_a^i - T_w^i)}{\frac{G}{\mathcal{E}}(C_a + C_v s^i) + C_v} \quad (3)$$

where  $\lambda$  is the latent heat of evaporation of the liquid and  $C_v(T_a^i - T_w^i)$  is the heat energy for superheating the saturated vapor from  $T_w^i$  to the air temperature  $T_a^i$ . For most common liquids and ordinary air temperature ( $< 200^\circ\text{C}$ ) this superheating energy is small, by about two orders of magnitude, compared to  $\lambda$ . However, for very hot and dry air conditions, this term begins to be important.

Thus we define dimensionless heat- and mass-transfer coefficients as follows:

$$j_h = \frac{h}{GC_a} \quad (6a)$$

$$j_m = \frac{h_m \rho_a^{\text{mean}}}{G} \quad (6b)$$

where  $\rho_a^{\text{mean}}$  is the dry air density at the arithmetic mean temperature of the inlet and outlet air temperatures. Knowing the evaporation rate  $\mathcal{E}$ , the conditions of the air at inlet and the vapor properties, the transfer coefficients can be calculated from equations 6(a) and 6(b). The heat-transfer coefficients obtained from equa-

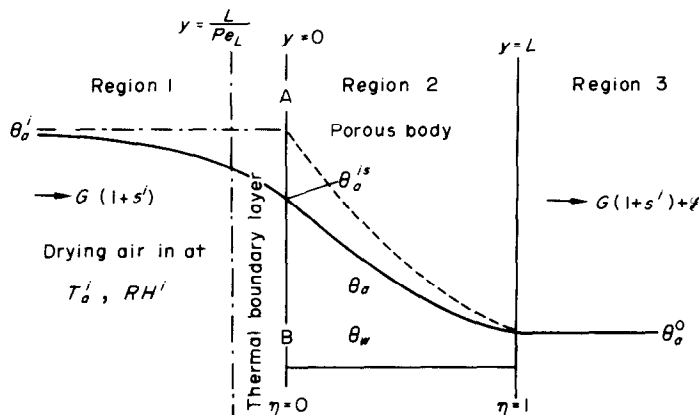


FIG. 3. Model for bulk analysis including the effects of thermal conduction in the air stream.

We now define the transfer coefficients (based on unit nominal area) as follows:

$$\text{Heat-transfer coefficient} = h = \frac{\lambda + C_v(T_a^i - T_w^i)}{\overline{(T_a - T_w)}} \quad (4a)$$

$$\text{Mass-transfer coefficient} = h_m = \frac{\mathcal{E}}{\overline{(\rho_{v,w} - \rho_{v,a})}} \quad (4b)$$

where the "bars" on the differences of temperature and vapor density indicate some suitably defined mean values. The value of the transfer coefficients obtained depends therefore on the definition of the "mean" used. Since the problem is similar to the heat exchanger problem, we define them as logarithmic means as follows:

$$\overline{(T_a - T_w)} = \frac{(T_a^i - T_a^o)}{\ln \left[ \frac{(T_a^i - T_w)}{(T_a^o - T_w)} \right]} \quad (5a)$$

$$\overline{(\rho_{v,w} - \rho_{v,a})} = \frac{(\rho_{v,w} - \rho_{v,a})^i - (\rho_{v,w} - \rho_{v,a})^o}{\ln \left[ \frac{(\rho_{v,w} - \rho_{v,a})^i}{(\rho_{v,w} - \rho_{v,a})^o} \right]} \quad (5b)$$

tion 6(a) using the experimental data are shown in Fig. 4(a). These results are compared later to those obtained by taking into account the finite thermal and mass diffusion in air.

*Effect of conduction and diffusion in the air stream direction.* When the body being dried is very thin, the temperature gradient within the body may become so steep that heat conducted in the air stream may become comparable to the energy of evaporation. Then we can no longer assume that the temperature of air at the inlet surface of the porous body is the same as the inlet air temperature (as recorded by a thermocouple far upstream from the porous body). A temperature drop and a corresponding diffusive rise in humidity occur in a layer immediately upstream of the drying body surface. If  $l_T$  and  $l_s$  are the thicknesses of these upstream regions for significant changes in the air stream temperature and humidity, respectively, then it can be shown by analysis of the diffusion equation that

$$l_T \sim O \left[ \frac{K_a}{GC_a} \right] \quad (7a)$$

and

$$l_s \sim O\left[\frac{D\rho_a^i}{G}\right] \quad (7b)$$

Since the total energy flux entering the porous body remains unchanged in spite of the considerations of longitudinal conduction and vapor diffusion in the air stream, the temperature and the humidity of the air at the exit from the body remain unaltered, for any given evaporation rate. These values are obtained from equations (1) and (3). The heat- and mass-transfer coefficients as defined in equation (4) have values that depend upon the upstream diffusion (of heat and mass) and hence are functions of the Peclet number ( $Pe_L$ ) and Lewis number. Coefficients not so dependent upon these numbers could be defined in terms of the con-

ditions of air at the porous body surfaces. Thus we redefine the transfer coefficients of equations (4) by using the temperature and vapor density differences at the inlet surface of the porous body instead of the upstream conditions.

By solving diffusion equations in the air ahead of the body (with logarithmic variations of temperature and humidity inside of the body)\* the air temperature  $T_a^{is}$  and the humidity  $s_a^{is}$  at the position of the inlet surface of the body can be shown to be

$$T_a^{is} = \frac{\varepsilon T_a^0 + 0.632 T_a^i Pe_L}{\varepsilon + 0.632 Pe_L} \quad (8a)$$

and

$$s_a^{is} = \frac{\varepsilon s^0 + 0.632 s^i Le Pe_L}{\varepsilon + 0.632 Le Pe_L} \quad (8b)$$

where  $\varepsilon$  is the porosity of the body during constant drying conditions.

Using the above relations and equations (5) and (6) and defining the mean temperature and vapor concentration differences with the values of temperature and humidity at the inlet surface of the body, the transfer coefficients  $j_h$  and  $j_m$  are calculated from the experimental measurements. The values of  $j_h$  obtained without consideration of the longitudinal diffusion in air are shown in Fig. 4(a) and those obtained after including the effects of conduction and diffusion in the air stream are shown in Fig. 4(b).  $j_m$  is shown plotted in Fig. 5. The Peclet numbers (for correlating the

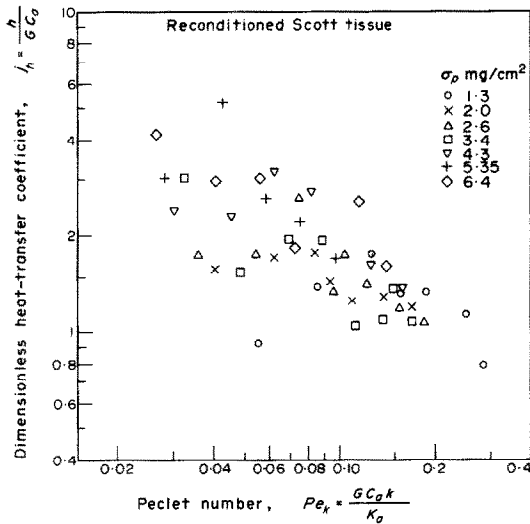


FIG. 4(a). Heat-transfer coefficient calculated using the bulk analysis without considering the thermal conduction and vapor diffusion in the air stream.

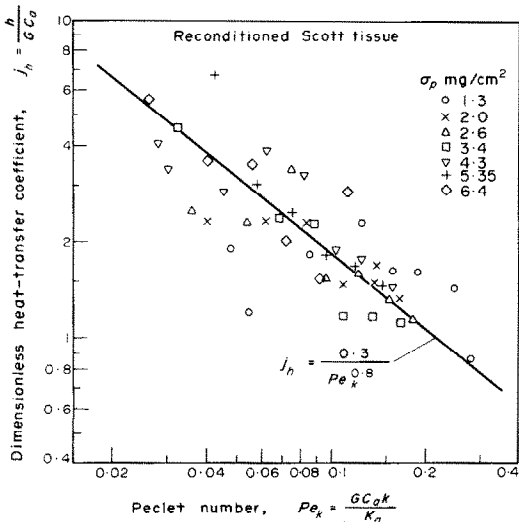


FIG. 4(b). Heat-transfer coefficient calculated using the bulk analysis including the effects of thermal conduction and vapor diffusion in the air stream.

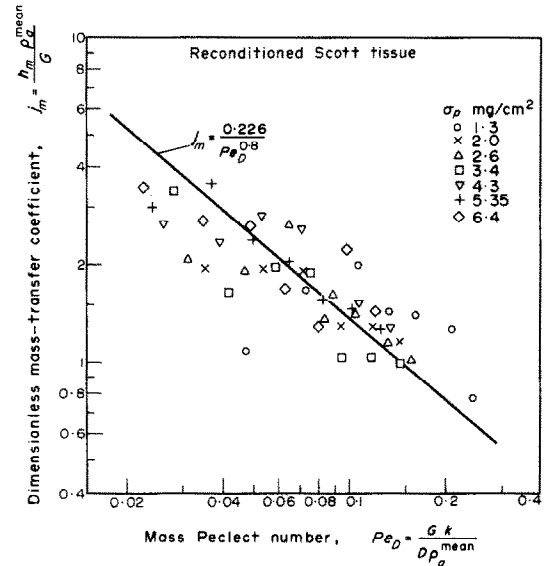


FIG. 5. Mass-transfer coefficient calculated using the bulk analysis including the effects of thermal conduction and vapor diffusion in the air stream.

transfer coefficients) are defined with the "Darcy Permeability" of dry sheets of paper as the characteristic dimension. This permeability is not only a function of the fiber dimension and the porosity but

\*Logarithmic profiles result as a consequence of the assumption that the rate of exchange of a property is proportional to the difference of its value in the air and its constant value in the solid body.

also of the fiber size distribution. Hence it is a unique property of the porous body (for details see Spielman [9], Beavers and Sparrow [10]).

The correlations obtained are compared in Fig. 6 with the data of London *et al.* [11] for turbulent flow of air over randomly arranged crossed rod matrices.

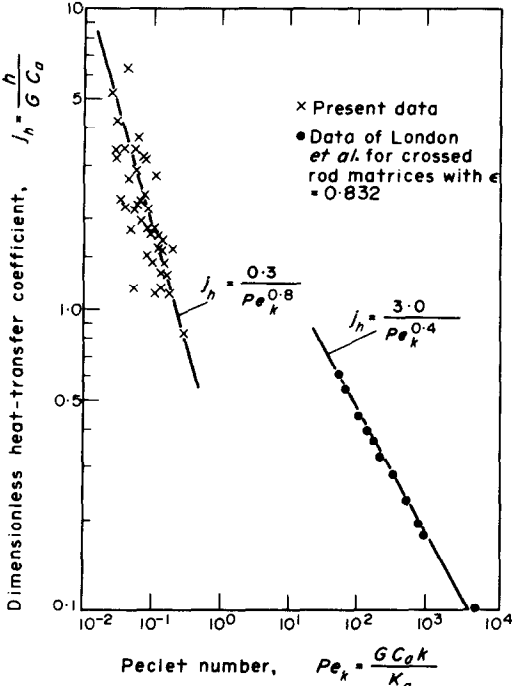


FIG. 6. Comparison of the present data and the data of London *et al.*, for flow through crossed rod matrices.

That the air flow through the paper sheets in this study is laminar is clearly seen. The value of  $-0.8$  for the slope in the laminar region is close to the slope values (between  $-0.7$  and  $-1.0$ ) published in the heat exchanger literature, for the flow of gases inside tubes [12]. Comparison of Figs. 4(a) and (b) indicates that for Peclet numbers ( $Pe_k$ ) less than about 0.1 the effect of the longitudinal conduction in the stream is felt. By correcting for this effect, the calculated heat-transfer coefficient becomes higher and correlates a little better.

So far, we have used the experimental data and a simple theory based on just the conservation laws and constant material properties (which result in logarithmic variations of the temperature and humidity within the porous body) to correlate the transfer coefficients with Peclet number characterizing the flow and the nature of the porous body. The utility of this correlation is clearly seen in obtaining the evaporation rate from any porous body given the flow rate, the conditions of the air, and the characteristics of the body (in terms of dry Darcy Permeability). For example, once the transfer coefficients are obtained, then the outflow conditions of the air can be calculated from equations 4(a) and (b) by a method of trial and error. In the analysis that follows, a more detailed analysis of the transfer process within the porous body is considered and an expression derived for the total evaporation

rate as a function of transfer coefficient (and therefore of flow rate).

2. General equations

(i) *Constant rate analysis (time independent formulation).* In this section we formulate the steady state problem of evaporation within the porous body and integrate the equations to obtain the constant drying rate. We assume that at any point within the porous body the solid and liquid temperatures are the same and equal to the local wet bulb temperature corresponding to the conditions of the air at that point. We also assume that there is no liquid movement in the solid matrix. The following nondimensional form of conservation equations apply (see Fig. 7) for evap-

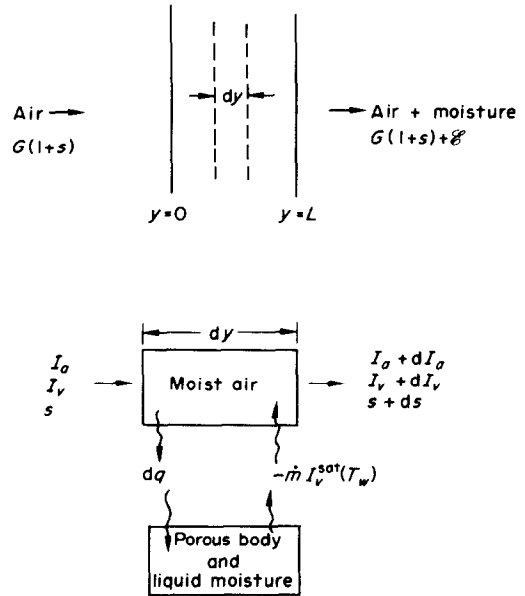


FIG. 7. Model for the derivation of the general equations for drying.

oration within the porous body.

Vapor conservation in the air stream

$$\frac{dQ}{d\eta} + \left[ (1 + \beta_v s) - \frac{\epsilon \beta_v}{Le Pe_L} \frac{ds}{d\eta} \right] \frac{d\theta_a}{d\eta} - f \beta_v (\theta_a - \theta_w) - \frac{\epsilon}{Pe_L} \frac{d^2 \theta_a}{d\eta^2} = 0. \quad (9)$$

Energy equation for the air stream

$$\frac{ds}{d\eta} + f - \frac{c}{Pe_L Le} \frac{d^2 s}{d\eta^2} = 0. \quad (10)$$

Vapor liberation from the liquid

$$\frac{dQ}{d\eta} + f = 0. \quad (11)$$

Heat-transfer rate equation

$$\frac{dQ}{d\eta} = j_h (\theta_a - \theta_w). \quad (12)$$

Mass-transfer rate equation

$$\dot{f} = -j_m \left[ \frac{\left[ A \exp\left(\frac{-B}{\theta_w}\right) \right] (\mu + s)}{(\theta_w/\theta_a)} - s \right] \quad (13)$$

The above equations are applicable within the porous body, i.e. in the region  $0 < \eta < 1$ .

In the above derivation  $j_h$  and  $j_m$  are the non-dimensional heat- and mass-transfer coefficients defined by

$$j_h = \frac{HL}{GC_a} \quad (14a)$$

$$j_m = \frac{H_m L}{G} \rho_a^{\text{mean}} \quad (14b)$$

where  $H$  and  $H_m$  are the volumetric transfer coefficients (based on a unit volume of the porous body) for transfer of heat and mass respectively within the porous body between air and the solid structure. Also assumed in the above derivation is the applicability of Clausius Clayperon equation for the saturated vapor pressure over the moisture surface within the porous body.

The relation between these volumetric transfer coefficients and the area coefficients defined in Section 1 (equations 4) can be established if we assume that these volumetric coefficients are constant throughout the thickness of the porous body. Then,

$$h \simeq HL \frac{\frac{1}{L} \int_{y=0}^L (T_a - T_w) dy}{(T_a - T_w)} \quad (15)$$

and similarly for the mass-transfer coefficient. The term  $\frac{1}{L} \int_{y=0}^L (T_a - T_w) dy$  is defined in equation 5(a). It has been shown elsewhere (8) that the value of the arithmetic average in the numerator, and log mean in the denominator of the above expression are almost equal, and  $h$  and  $H$  are proportional.

For the temperature and humidity variations in the air outside of the porous body we write the following diffusion equations,

$$\frac{ds}{d\eta} - \frac{1}{Pe_L Le} \frac{d^2s}{d\eta^2} = 0 \quad (16)$$

$$\left[ (1 + \beta_v s) - \frac{\beta_v}{Pe_L Le} \frac{ds}{d\eta} \right] \frac{d\theta_a}{d\eta} = \frac{1}{Pe_L} \frac{d^2\theta_a}{d\eta^2} \quad (17)$$

in the region  $-\infty < \eta < 0$  and  $1 < \eta < \infty$ .

The set of equations (9)–(13) and (16) and (17) are linked by the boundary conditions at  $\eta = 0$  and  $\eta = 1$ . Physically these boundary conditions express the continuity of the functions (temperature and humidity) and the thermal and vapor flux equalities. Explicitly these are,

$$\eta = -\infty \quad \begin{matrix} \theta_a = \theta_a^i \\ s = s^i \end{matrix} \quad \text{inlet conditions}$$

$$\eta = 0 \quad \begin{matrix} (\theta_a)_{\eta=0^-} = (\theta_a)_{\eta=0^+} \\ (s)_{\eta=0^-} = (s)_{\eta=0^+} \end{matrix}$$

$$\left( \frac{d\theta_a}{d\eta} \right)_{\eta=0^-} = \left( \frac{d\theta_a}{d\eta} \right)_{\eta=0^+} \quad (18)$$

$$\left( \frac{ds}{d\eta} \right)_{\eta=0^-} = \left( \frac{ds}{d\eta} \right)_{\eta=0^+}$$

$\eta = 1$  Conditions are similar to that at  $\eta = 0$ , indicating the equality of functions and the fluxes on both sides of the surface of the body.

$$\eta = +\infty \quad \begin{matrix} \theta_a = \text{finite} \\ s = \text{finite} \end{matrix}$$

The unknowns in the problem are  $s, \theta_a, \theta_w, f$  and  $Q$ . In principle at least it is possible to solve the equations, since there are enough equations and conditions.

Equations (16) and (17) have exact solutions, consistent with the boundary conditions at  $\eta = \pm\infty$ . These solutions indicate that the temperature and humidity remain constant down stream of the porous body and in front of the body they decrease exponentially. Because of the nonlinear terms in the equations (9)–(13), simple analytical solutions for them are impossible. However, under some simplifying assumptions fairly good approximate solutions are obtained. These assumptions are,

1. The heat of superheating the vapor from the wet bulb temperature to the air stream temperature is small compared to the latent heat of vaporization;
2. The sensible heat change of the moist air can be approximated by the sensible heat change of the dry air alone;
3. A linear Clausius–Clayperon equation is used.
4. The Lewis number is unity.

Based on the above approximations, the variations of air temperature and humidity within the porous body is,

$$\frac{T_a^i - T_a}{T_a^i - T_w^i} = \frac{\lambda}{C_a} \frac{s - s^i}{T_a^i - T_w^i} = \begin{cases} \delta \left[ 1 - \frac{2}{(1 + D_2 \Delta)} \exp\left(\frac{Pe_L \eta}{2\varepsilon}\right) \left\{ \cosh\left(\frac{Pe_L \eta \Delta}{2\varepsilon}\right) - D_2 \sinh\left(\frac{Pe_L \eta \Delta}{2\varepsilon}\right) \right\} \right] & \text{in the region } 0 \leq \eta \leq 1 \quad (19a) \end{cases}$$

$$\delta \left[ 1 - \frac{2}{(1 + D_2 \Delta)} \right] \exp(Pe_L \eta) \quad \text{in the region } -\infty \leq \eta \leq 0 \quad (19b)$$

where,

$$\delta = \frac{\left(b + \frac{j_h}{j_m}\right)}{(b+1)}$$

$$\Delta = \sqrt{\left(1 + \frac{4j_h \varepsilon}{\delta Pe_L}\right)} \quad (20)$$

$$D_2 = \frac{\cosh\left(\frac{Pe_L \Delta}{2\varepsilon}\right) + \Delta \sinh\left(\frac{Pe_L \Delta}{2\varepsilon}\right)}{\sinh\left(\frac{Pe_L \Delta}{2\varepsilon}\right) + \Delta \cosh\left(\frac{Pe_L \Delta}{2\varepsilon}\right)}$$

and "b" is defined by the linearized Clausius-Clayperon equation,

$$p^{\text{sat}}(T_w) = a + b \left(\frac{C_a}{\lambda \mu}\right) T_w. \quad (21)$$

Numerical solution to the set of equations (9)–(17) for the general case is complicated. For non-unity Lewis numbers this is a two point, two boundary search problem. The special case of  $Pe_L \rightarrow \infty$  was solved (numerically), because in this case there is negligible longitudinal conduction and diffusion and hence the problem is simpler than the general one. The solution obtained was within 2 per cent of the solution given in equations (19) and (23), except for the case of high air temperature and humidity ( $> 200^\circ\text{C}$ , 30% R.H.) indicating that under these conditions some of the assumptions (mainly the assumption that the wet air sensible heat change = dry air sensible heat change) used in the analytical derivations are no longer valid. The results presented in Figs. 8 and 9 are however adequate for many engineering purposes.

Using the experimental coefficient of evaporation and its relationship with the heat-transfer coefficient

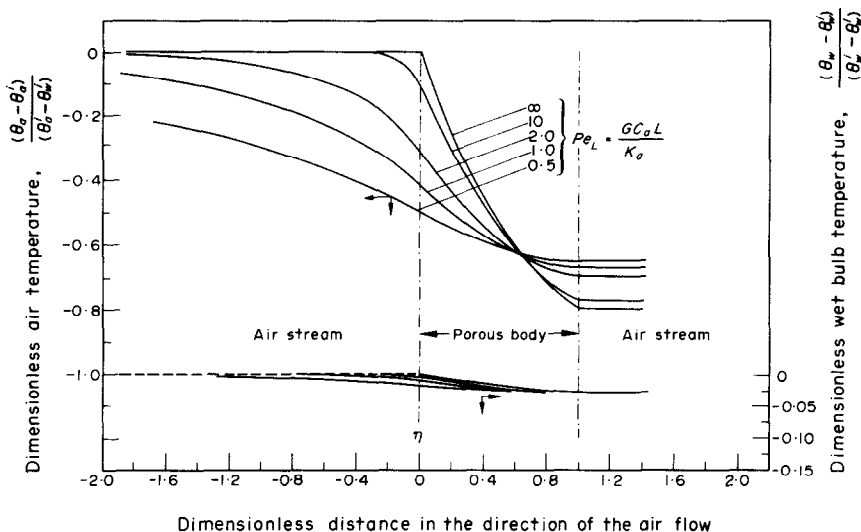


FIG. 8. Air temperature and wet bulb temperature variation through the porous body in the constant drying period.

We now define a coefficient of evaporation ( $E$ ) which represents the efficiency of evaporation. Physically, it is the ratio of evaporation to the theoretical maximum evaporation that can be obtained corresponding to the flow rate and inlet conditions of the air. That is,

$$E = \frac{\varepsilon \lambda}{GC_a(T_a^i - T_w^i)}. \quad (22)$$

Thus by the solution of the equations (9)–(13) and using (22) and (1)

$$E = \delta \left[ 1 - \frac{2}{(1 + D_2 \Delta)} \exp\left(\frac{Pe_L}{2\varepsilon}\right) \times \left\{ \cosh\left(\frac{Pe_L \Delta}{2\varepsilon}\right) - D_2 \sinh\left(\frac{Pe_L \Delta}{2\varepsilon}\right) \right\} \right]. \quad (23)$$

Results of equations (19) and (23) are shown in Figs. 8 and 9 respectively for various values of the flow Peclet numbers. These figures aid in the evaluation of the evaporation rate  $\varepsilon$  for any given flow condition.

(equation 23), the latter were calculated. These values are shown in Fig. 4(b). The agreement between the values of the transfer coefficients obtained by the detailed analysis methods and bulk analysis method, justifies the use of the log mean temperature and vapor density differences in the latter.

(ii) *Falling rate analysis (non-steady formulation)*. So far only the quasi-steady state portion of the drying phenomenon was considered. Experimentally, it was observed that when the mean moisture content of the paper sheets was between 100 and 80 per cent, the rate of drying started to decrease.

The decrease in the rate of drying, at low moisture contents can be caused by either or both of the following:

1. Decrease in the vapor concentration over the "surface" of the fibers (in effect it is the decrease of the partial pressure of the vapor compared to the saturated vapor pressure). This phenomenon



could be represented by (see Prahl [11]).

$$p_v = p_v^{sat}(T_s) \times r(f, T_s). \quad (24)$$

2. Decrease in the "liquid" area exposed for evaporation as a result of the recession of moisture into smaller and smaller cavities, together with the fiber shrinkage.

In deriving the falling rate equations both of the above phenomena are included. The latter effect is included in the changing values for the transfer coefficients. Assuming the heat of vaporization to be independent of the moisture content and the vapor pressure ratio ( $r$  in equation 24) to be independent

Mass-transfer rate; between air and body

$$\frac{\partial f}{\partial \tau} = -j_m \left\{ \frac{r(f, \theta_s) \left\{ A \exp\left(\frac{-B}{\theta_s}\right) \right\} (\mu + s)}{\left(\frac{\theta_s}{\theta_a}\right)} - s \right\} \quad (29)$$

for the region  $0 \leq \eta \leq 1$ .

These equations are complemented with vapor and thermal diffusion equations (16) and (17) for the air flow region external to the body. The boundary conditions are the same as specified in equation (18) except that all the total derivatives have to be replaced by

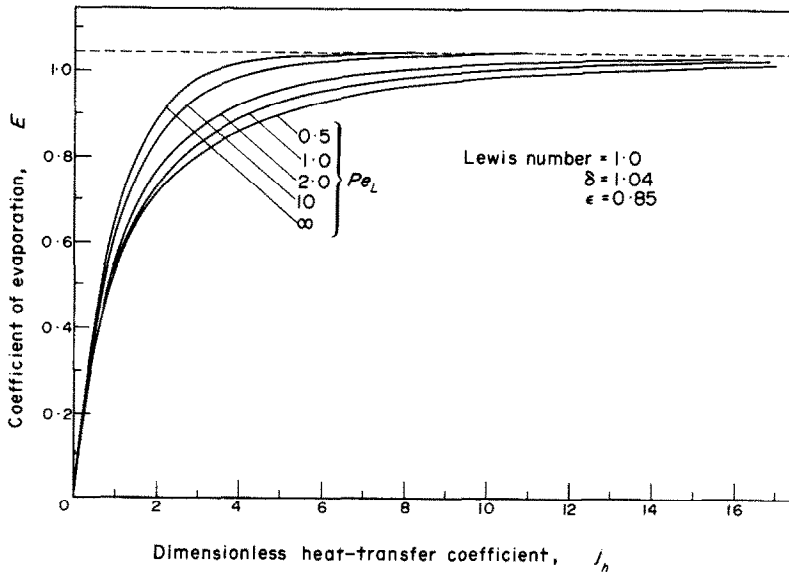


FIG. 9. Coefficient of evaporation as a function of the heat-transfer coefficient and the Peclet number based on the thickness of the sheet.

of the temperature, we write the following equations for the temporal and spatial variation of air temperature, humidity and moisture content in the porous body; Vapor flow in air stream

$$\frac{\partial s}{\partial \eta} + \frac{\partial f}{\partial \tau} - \frac{\epsilon}{Pe_L Le} \frac{\partial^2 s}{\partial \eta^2} = 0. \quad (25)$$

Air side energy conservation

$$\frac{\partial Q}{\partial \eta} + \left[ (1 + \beta_v s) - \frac{\beta_v}{Pe_L Le} \frac{\partial s}{\partial \eta} \right] \frac{\partial \theta_a}{\partial \eta} - \beta_v (\theta_a - \theta_s) \frac{\partial f}{\partial \tau} - \frac{\epsilon}{Pe_L} \frac{\partial^2 \theta_a}{\partial \eta^2} = 0. \quad (26)$$

Body side energy conservation

$$\frac{\partial Q}{\partial \eta} + \frac{\partial f}{\partial \tau} - (\beta_s + \beta_w f) \frac{\partial \theta_s}{\partial \eta} + \frac{(1 - \epsilon)}{Pe_s} \frac{\partial^2 \theta_s}{\partial \eta^2} = 0. \quad (27)$$

Heat-transfer rate; between air and body

$$\frac{\partial Q}{\partial \eta} = j_h (\theta_a - \theta_s). \quad (28)$$

partial derivatives with respect to the spatial coordinate  $\eta$ . Assuming that there is no external heat addition to the solid matrix as such, we have the boundary condition for the temperature of the solid

$$\frac{\partial \theta_s}{\partial \eta} = 0 \quad \text{for } \eta = 0, 1 \quad \text{at all times.} \quad (30)$$

The initial conditions are specified from the solutions of the constant rate drying problem. The initial conditions are the values that exist when the leading edge moisture value is "critical". The value of the critical moisture is that value below which the vapor pressure ratio (that is relative humidity over the fibers) starts to change significantly from unity. This vapor pressure moisture history is determined experimentally. In this paper critical moisture content value is assumed to be 0.4, based on the results of separate experiments to determine the desorption characteristics of the tissue papers used.

In the body side energy equation, terms arising from the heating of the solid matrix and the thermal con-

duction in the solid fibers, caused by the solid temperature gradient, are included. However, no provision is made for any possible moisture movement caused by moisture gradients. It is assumed that the moisture has no interconnecting continuous liquid paths within the porous matrix. The vapor pressure ratio  $r$  in equation (29) depends on the moisture content, temperature and nature of fibers (Prahl [13]). For obtaining the explicit relationships between  $r$  and the moisture fraction, separate experiments were conducted. Characteristic hysteresis curves for the absorption and desorption were obtained. For the drying calculations the results of the desorption characteristic were used. The following equation accurately represents the

where  $j_n$  in the denominator of the left hand side is the constant drying rate transfer coefficient.

Equation (31) is derived from assuming a linear relation for the effective thermal conductivity of the wet solid. It is assumed that beyond 100 per cent moisture the conductivity is essentially that of water. Equation (32), which shows the variation of the local heat-transfer coefficient (and similarly the local mass-transfer coefficient) with the local moisture content, is derived by assuming that the coefficients vary with the diameter of the fiber according to McAdams [14] correlation (for flow over an isolated cylinder) and that the swollen fiber volume is equal to the sum of the volumes of the dry fiber and the moisture it contains.

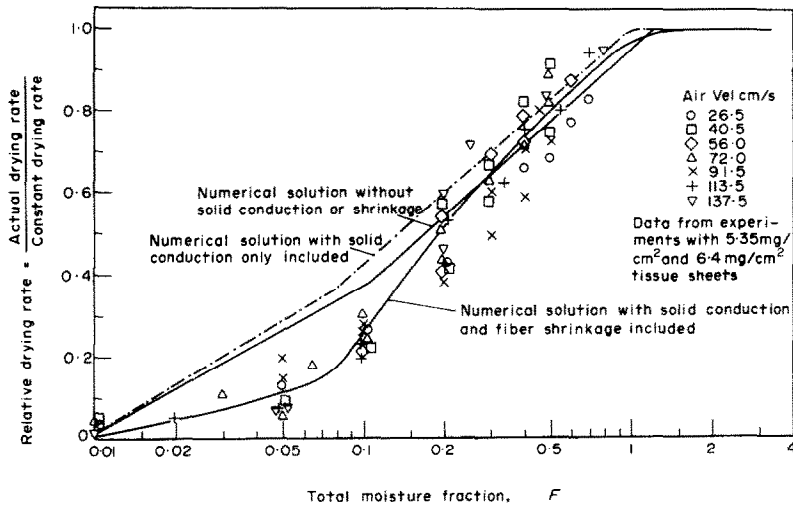


FIG. 10. Drying rate in the falling rate period vs sheet moisture fraction.

experimentally observed desorption characteristic for the tissue paper used in the drying tests;

$$r = \begin{cases} 1.0 & \text{for } f > 0.4 \\ 1 - \exp[-14.5(f-0.4)] & \text{for } 0.07 < f < 0.4 \\ 51.29f^{1.855} & \text{for } f < 0.07. \end{cases}$$

Equations (25)–(29) together with (16) and (17) form a coupled set of partial, nonlinear differential equations. As before the solution to the general problem is difficult and therefore a simplified problem of  $Pe_L \rightarrow \infty$  is solved. This assumption essentially eliminates the longitudinal thermal conduction and vapor diffusion in the air stream. However, the thermal conduction in the solid matrix and the area change caused by fiber shrinkage are retained. These are incorporated by using the following equations [8] for the effect of moisture on the thermal conductivity of the solid fibers and the variation of transfer coefficient [as defined in equations 14(a) and 14(b)] with change in fiber area per unit volume of the body.

$$K_S^{\text{eff}}(f) = \begin{cases} [K_S + (K_w - K_S)f] & \text{for } f < 1 \\ K_w(1 - \epsilon) & \text{for } f > 1 \end{cases} \quad (31)$$

and

$$\frac{j_n(f)}{j_n} = \left[ \frac{(1 + 1.55f)}{(1 + 1.55 \times 1.5)} \right]^{0.165} \quad (32)$$

#### NUMERICAL SOLUTIONS

The set of equations presented above were solved by the method of finite differences by marching in time through a few steps of the constant drying rate period and then through the falling rate period. In order to obtain a stable numerical procedure for large time steps, the solid temperature is solved from equation (27) by writing the temporal differentials of  $\theta_s$  in the implicit form.

The algorithm for solution used an iterative convergence scheme. That is, the calculations were repeated at each time step until the drying rates agreed to within 5 per cent at every spatial grid point (within the porous body). Many other features used in the computation scheme are those discussed by Lapidus [13] for solving partial differential equations.

#### RESULTS

The results obtained by the numerical solution of the set of equations (25)–(29) are presented in Fig. 10 together with experimental data for thick sheets and high flow rates ( $Pe_L$  large). The ordinate (relative drying rate) is the ratio of the actual drying rate to the constant drying rate. It is seen from the comparison that the theoretical model presented agrees well with data. While the change in thermal conduction and the decrease in transfer area have considerable influence

on the drying rate, the principal mechanism of decrease is the vapor pressure change with moisture content.

The results of falling rate analysis can therefore be correlated by

Relative drying rate

$$= \begin{cases} 1.0 & \text{for } F > 0.9 \\ 0.34 \ln(F) + 1.035 & \text{for } 0.1 < F < 0.9 \\ 0.087 \ln(F) + 0.45 & \text{for } F < 0.1. \end{cases} \quad (33)$$

#### DISCUSSIONS

The models presented describe the phenomenon of evaporation from inside a transpiration dried porous body. The approach taken is similar to the analysis of the transfer phenomenon in heat/mass exchangers. The bulk analysis presented is useful for expressing the experimental data in a unified way in terms of heat and mass transfer coefficients. It has been shown that a more refined analysis for the constant drying problem leads essentially to the same results, indicating thereby, that the use of log-mean temperature and vapor concentration differentials are sufficiently accurate to correlate the information.

The scatter of the data presented in Figs. 4(a), 4(b), and 5 does not represent the actual scatter in the experimentally observed drying rates. The reason for this is simple. At low air velocities (i.e. low Peclet number  $Pe_k$ ) the condition of the air at the outlet end of the porous body is close to saturation and its temperature near the wet bulb temperature. To measure these small differences is impractical. In these experiments, outlet properties were computed from the inlet conditions and the measured drying rates. Thus, small errors in these latter measurements produce large percentage errors in the outlet differences. Because of this, the log-mean differences change considerably, resulting in large changes in the calculated values of the transfer coefficients. In fact, the largest scatter in the heat-transfer coefficient  $j_h$  is about 87 per cent, which represents a 4.8 per cent error in the drying rate.

The capability of air to dry a certain porous body depends, as expected, on its "dryness" and temperature. This fact could be utilized in defining an "evaporative potential" for air which is essentially the difference between the dry and wet bulb temperatures. Given this evaporative potential, we have provided a means of estimating the constant drying rate once the flow conditions and the body characteristics are known. This is expressed as a "coefficient of evaporation" (see Fig. 8). Once the flow conditions are known, the heat-transfer coefficient is determined from Fig. 4 and the coefficient of evaporation  $E$  from Fig. 8. Then using the correlation equation (33), the drying rate at any moisture content can be estimated. It is then a trivial matter to estimate the drying time from any moisture content to any other lower moisture content.

It has become clear from the analysis and solution of the models presented and their comparison with experimental data that for low velocities and thin sheets (i.e.  $Pe_k < 0.1$ ) the effects of thermal and vapor diffusion in the air stream (in the direction of stream flow) play

an important role in the drying process. Also, it is seen that the primary cause of the decrease in the drying rate is the hygroscopicity (with its effect on vapor pressure) though change of fiber dimensions also have a marked effect. A simple order-of-magnitude estimate of the time of diffusion of moisture out of the fibers in the "radial" direction indicates that diffusion of moisture through the solid wall of the fiber is not the rate controlling the phenomenon during the falling rate period.

The results obtained have been used to predict the drying times of paper under the conditions of experiments reported by Walser and Swenson [13]. The results of their experiments in terms of drying time and the present theoretical prediction agree within 7 per cent.

#### CONCLUSIONS

1. Experimental heat- and mass-transfer coefficients have been correlated against flow Peclet number based on the dry Darcy permeability of the porous body as suggested by a simple analysis of the drying process.
2. At low flow rate and for thin bodies, the effects of diffusion of vapor and heat in the air stream are important.
3. The principal cause by which the rate of drying decreases is the hygroscopicity of the paper. However, the dimension change due to shrinkage also has considerable effect on the drying.
4. The correlations presented are in nondimensional form in terms of an evaporative coefficient and relative drying rate. These are useful for predicting drying times under different air temperatures, humidities, and flow rates.

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